Oxford Cambridge and RSA

Accredited

AS Level Further Mathematics A
Y533 Mechanics
Sample Question Paper Version 2

Date – Morning/Afternoon
Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\ \text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

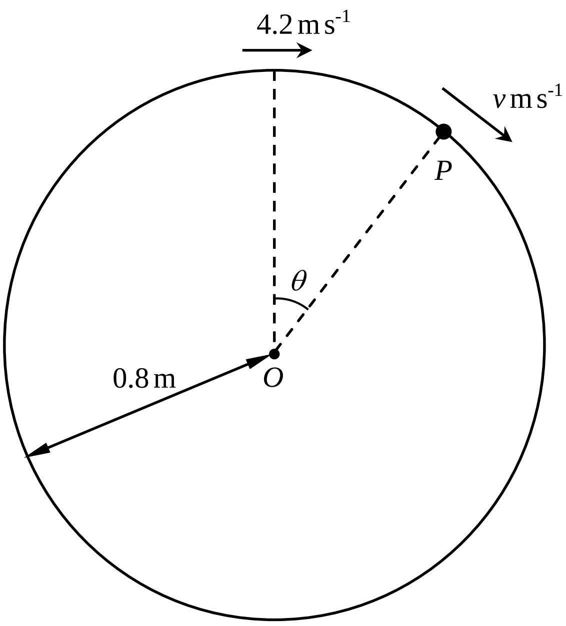
Answer **all** the questions.

1 A roundabout in a playground can be modeled as a horizontal circular platform with centre *O*. The roundabout is free to rotate about a vertical axis through *O*. A child sits without slipping on the roundabout at a horizontal distance of 1.5 m from *O* and completes one revolution in 2.4 seconds.

(i) Calculate the speed of the child. **[3]**

(ii) Find the magnitude and direction of the acceleration of the child. **[3]**

2



A smooth wire is shaped into a circle of centre *O* and radius 0.8 m. The wire is fixed in a vertical plane. A small bead *P* of mass 0.03 kg is threaded on the wire and is projected along the wire from the highest point with a speed of $4.2\ \mathrm{m\,s^{-1}}$. When *OP* makes an angle θ with the upward vertical the speed of *P* is $v\ \mathrm{m\,s^{-1}}$ (see diagram).

(i) Show that $v^2 = 33.32 - 15.68\cos\theta$. **[4]**

(ii) Prove that the bead is never at rest. **[1]**

(iii) Find the maximum value of *v*. **[2]**

3 **(i)** Write down the dimension of density. **[1]**

The workings of an oil pump consist of a right, solid cylinder which is partially submerged in oil. The cylinder is free to oscillate along its central axis which is vertical. If the base area of the pump is 0.4 m^2 and the density of the oil is 920 kg m^{-3} then the period of oscillation of the pump is 0.7 s.
A student assumes that the period of oscillation of the pump is dependent only on the density of the oil, ρ, the acceleration due to gravity, g, and the surface area, A, of the circular base of the pump. The student attempts to test this assumption by stating that the period of oscillation, T, is given by $T = C\rho^{\alpha} g^{\beta} A^{\gamma}$ where C is a dimensionless constant.

(ii) Use dimensional analysis to find the values of α, β and γ. **[4]**

(iii) Hence give the value of C to 3 significant figures. **[2]**

(iv) Comment, with justification, on the assumption made by the student that the formula for the period of oscillation of the pump was dependent on only ρ, g and A. **[2]**

4 A car of mass 1250 kg experiences a resistance to its motion of magnitude kv^2 N, where k is a constant and v m s^{-1} is the car's speed. The car travels in a straight line along a horizontal road with its engine working at a constant rate of P W. At a point A on the road the car's speed is 15 m s^{-1} and it has an acceleration of magnitude 0.54 m s^{-2}. At a point B on the road the car's speed is 20 m s^{-1} and it has an acceleration of magnitude 0.3 m s^{-2}.

(i) Find the values of k and P. **[7]**

The power is increased to 15 kW.

(ii) Calculate the maximum steady speed of the car on a straight horizontal road. **[3]**

5

The masses of two spheres A and B are $3m$ kg and m kg respectively. The spheres are moving towards each other with constant speeds $2u\,\mathrm{m\,s^{-1}}$ and $u\,\mathrm{m\,s^{-1}}$ respectively along the same straight line towards each other on a smooth horizontal surface (see diagram). The two spheres collide and the coefficient of restitution between the spheres is e. After colliding, A and B both move in the same direction with speeds $v\,\mathrm{m\,s^{-1}}$ and $w\,\mathrm{m\,s^{-1}}$, respectively.

(i) Find an expression for v in terms of e and u. **[6]**

(ii) Write down unsimplified expressions in terms of e and u for

(a) the total kinetic energy of the spheres before the collision, **[1]**

(b) the total kinetic energy of the spheres after the collision. **[2]**

(iii) Given that the total kinetic energy of the spheres after the collision is λ times the total kinetic energy before the collision, show that

$$\lambda = \frac{27e^2 + 25}{52}.$$

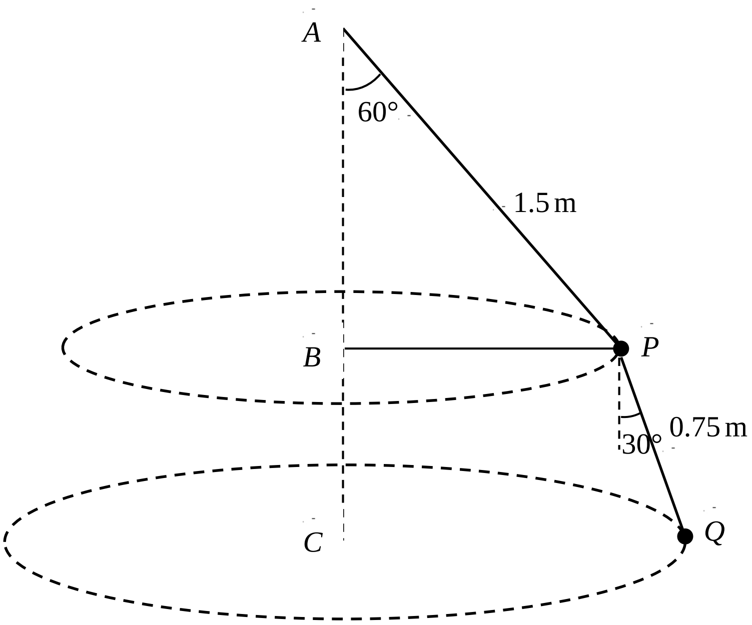
[3]

(iv) Comment on the cases when

(a) $\lambda = 1$,

(b) $\lambda = \frac{25}{52}$. **[3]**

6



The fixed points *A*, *B* and *C* are in a vertical line with *A* above *B* and *B* above *C*. A particle *P* of mass 2.5 kg is joined to *A*, to *B* and to a particle *Q* of mass 2 kg, by three light rods where the length of rod *AP* is 1.5 m and the length of rod *PQ* is 0.75 m. Particle *P* moves in a horizontal circle with centre *B*. Particle *Q* moves in a horizontal circle with centre *C* at the same constant angular speed ω as *P*, in such a way that *A*, *B*, *P* and *Q* are coplanar. The rod *AP* makes an angle of 60° with the downward vertical, rod *PQ* makes an angle of 30° with the downward vertical and rod *BP* is horizontal (see diagram).

(i) Find the tension in the rod *PQ*. **[2]**

(ii) Find ω. **[3]**

(iii) Find the speed of *P*. **[1]**

(iv) Find the tension in the rod *AP*. **[3]**

(v) Hence find the magnitude of the force in rod *BP*.
Decide whether this rod is under tension or compression. **[4]**

END OF QUESTION PAPER

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